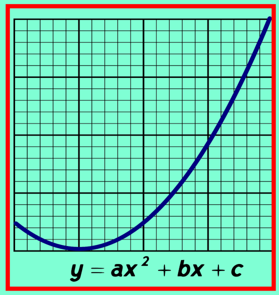
Math 125

Spring 2021

Lecture 20



Class QZ 15:

1) Write in $a+bi$ form: $\sqrt{18} - \sqrt{-100}$

$= \sqrt{9}\sqrt{2} - \sqrt{100}\sqrt{-1}$

$= \boxed{3\sqrt{2} - 10i}$

2) Simplify: $-2i(5+i) + 5(3+2i)$

$= \cancel{-10i} - 2i^2 + 15 \cancel{+10i} = -2(-1) + 15 = 2 + 15 = \boxed{17}$

3) Simplify: $(3+4i)^2 = (3+4i)(3+4i)$

$= 9 + 12i + 12i + 16i^2$

$= 9 + 24i + 16(-1)$

$= \boxed{-7 + 24i}$

4) Divide: $\frac{13}{2-3i}$

$= \frac{13(2+3i)}{(2-3i)(2+3i)} = \frac{13(2+3i)}{4 \cancel{+6i} \cancel{-6i} - 9i^2} = \frac{13(2+3i)}{4-9(-1)}$

$= \frac{13(2+3i)}{4+9} = \frac{\cancel{13}(2+3i)}{\cancel{13}}$

$= \boxed{2+3i}$

Simplify :

1) $\underbrace{(1+2i)(1-2i)}_{\text{Conjugates}}\underbrace{(3-4i)(3+4i)}_{\text{Conjugates}}$

$= \left[1 \cancel{-2i} \cancel{+2i} - 4i^2\right]\left[9 \cancel{+12i} \cancel{-12i} - 16i^2\right]$

$= \left[1 - 4(-1)\right]\left[9 - 16(-1)\right]$

$= \left[1 + 4\right]\left[9 + 16\right]$

$= 5 \cdot 25 = \boxed{125}$

Simplify

$(2-i)^3 = \underbrace{(2-i)(2-i)}(2-i)$

$= \left[4 - 2i - 2i + i^2\right](2-i)$

$= \left[4 \quad -4i + (-1)\right](2-i)$

$= (3-4i)(2-i)$

$= 6 - 3i - 8i + 4i^2$

$= 6 - 11i + 4(-1)$

$= \boxed{2 - 11i}$

Simplify:

1) $i^{45} = i^{44} \cdot i$

$= \left[i^{2}\right]^{22} \cdot i$

$= (-1)^{22}\, i$

$= 1\, i = \boxed{i}$

2) $i^{120} = \left(i^{2}\right)^{60}$

$= (-1)^{60}$

$= \boxed{1}$

3) $i^{78} = \left(i^{2}\right)^{39}$

$= (-1)^{39}$

$= \boxed{-1}$

4) $i^{91} = i^{90} \cdot i$

$= \left(i^{2}\right)^{45} \cdot i$

$= (-1)^{45} \cdot i$

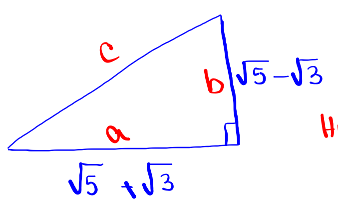
$= -1 \cdot i = \boxed{-i}$

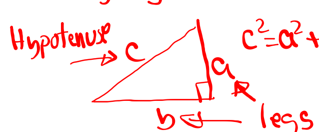
Find the missing Side:

Triangle

Right Triangle

Pythagorean Thrm





$c^2 = a^2 + b^2$

$c^2 = (\sqrt{5}+\sqrt{3})^2 + (\sqrt{5}-\sqrt{3})^2$

$= (\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{3}) + (\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})$

$= \sqrt{25} + \cancel{\sqrt{15}} + \cancel{\sqrt{15}} + \sqrt{9} + \sqrt{25} - \cancel{\sqrt{15}} - \cancel{\sqrt{15}} + \sqrt{9}$

$= 5 + 3 + 5 + 3 = 16 \Rightarrow c^2 = 16$

$\boxed{c = 4}$

Missing Side is 4

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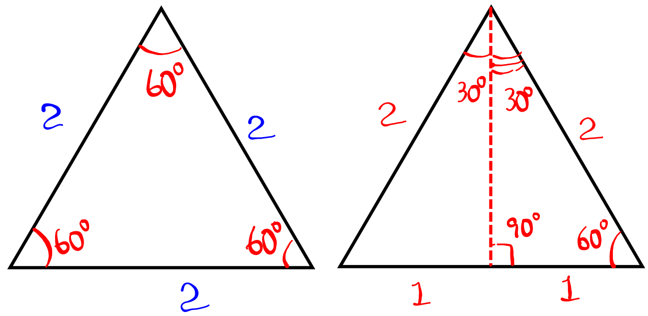
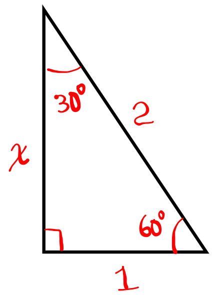
Special right Triangles:





Pythagorean thrm

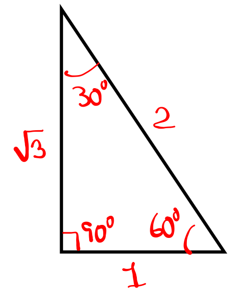
$x^2 + 1^2 = 2^2$

$x^2 + 1 = 4$

$x^2 = 4 - 1$

$x^2 = 3$

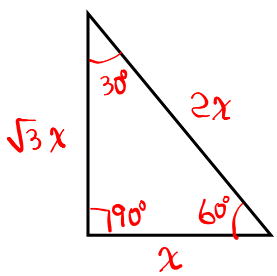
$\boxed{x = \sqrt{3}}$

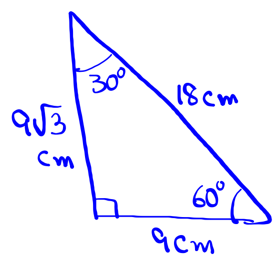


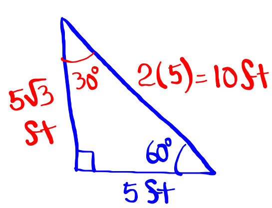
30°-60°-90° Right Triangle

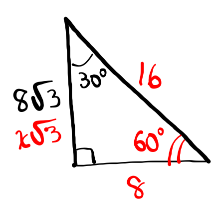
$2x = 18$

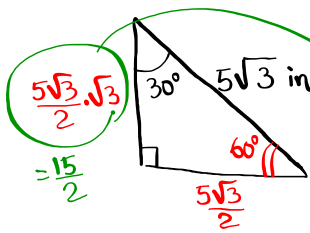
$x = 9$



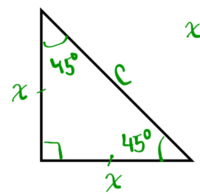








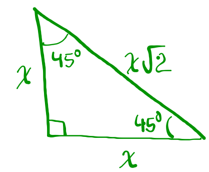
$\frac{5\sqrt{3}}{2} \cdot \sqrt{3} = \frac{5\sqrt{9}}{2} = \frac{5 \cdot 3}{2} = \frac{15}{2}$



$$x^2 + x^2 = c^2$$

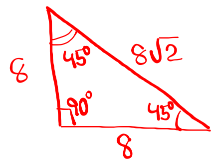
$$2x^2 = c^2$$

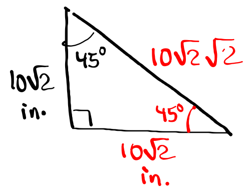
$$\boxed{c = x\sqrt{2}}$$



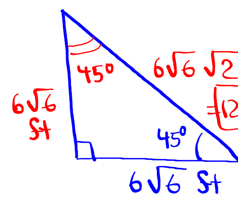
45°-45°-90°

Right Triangle





$$10\sqrt{2}\sqrt{2} = 10\sqrt{4} = 10 \cdot 2 = \boxed{20 \text{ in.}}$$



$$= \boxed{12\sqrt{3}}$$

$$6\sqrt{6}\sqrt{2} = 6\sqrt{12}$$

$$= 6\sqrt{4}\sqrt{3}$$

$$= 6 \cdot 2\sqrt{3} = \boxed{12\sqrt{3}}$$

Solve $\sqrt{3x - 10} - \sqrt{x+6} = 0$

Hint: Isolate one radical.

$$\sqrt{3x-10} = \sqrt{x+6}$$

$$\left(\sqrt{3x-10}\right)^2 = \left(\sqrt{x+6}\right)^2$$

$$3x - 10 = x + 6$$

$$3x - x = 6 + 10$$

$$2x = 16 \rightarrow \boxed{x = 8}$$

Check:

$$\sqrt{3(8) - 10} - \sqrt{8+6} = 0$$

$$\sqrt{24-10} - \sqrt{14} = 0$$

$$\sqrt{14} - \sqrt{14} = 0$$

$$0 = 0 \checkmark$$

$$\{ 8 \}$$

Solve $x - \sqrt{3x+10} = 0$

Hint: Isolate the radical

$x = \sqrt{3x+10}$

square both sides $(x)^2 = \left(\sqrt{3x+10}\right)^2$

$x^2 = 3x + 10 \rightarrow x^2 - 3x - 10 = 0$

$(x-5)(x+2) = 0$

$x - 5 = 0$ OR $x + 2 = 0$

$\boxed{x = 5}$ ✓ $\quad \cancel{x = -2}$ E.S.

$x - \sqrt{3x+10} = 0$

$5 - \sqrt{3(5)+10} = 0$
$5 - \sqrt{25} = 0$
$5 - 5 = 0$

$-2 - \sqrt{3(-2)+10} = 0$
$-2 - \sqrt{4} = 0$
$-2 - 2 = 0$
false

$\{5\}$

Solve $x + \sqrt{x-3} = 5$

Isolate the radical

$\sqrt{x-3} = 5 - x$

$\left(\sqrt{x-3}\right)^2 = (5-x)^2$

$x - 3 = (5-x)(5-x)$

$x - 3 = 25 - 5x - 5x + x^2$

$x - 3 = 25 - 10x + x^2$

$x^2 - 10x + 25 - x + 3 = 0$

$x^2 - 11x + 28 = 0 \Rightarrow (x-4)(x-7) = 0$

$x - 4 = 0 \quad x - 7 = 0$

$\boxed{x = 4}$ ✓ $\quad \cancel{x = 7}$ E.S.

Check

$x + \sqrt{x-3} = 5$

$4 + \sqrt{4-3} = 5$
$4 + \sqrt{1} = 5$
$4 + 1 = 5$
$5 = 5$ ✓

$7 + \sqrt{7-3} = 5$
$7 + \sqrt{4} = 5$
$7 + 2 = 5$
$9 = 5$
false

$\{4\}$

Solve $\sqrt[3]{2x+5}-1=2$

Hint: Isolate the radical

$$\sqrt[3]{2x+5}=3$$

$$\left(\sqrt[3]{2x+5}\right)^3=(3)^3$$

$$2x+5=27$$

$$2x=22 \quad \boxed{x=11}$$

$$\{11\}$$

Rationalize the denominator:

$$\frac{8}{\sqrt{6}}=\frac{8\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}}=\frac{8\sqrt{6}}{\sqrt{6^2}}=\frac{\cancel{8}^{4}\sqrt{6}}{\cancel{6}_{3}}=\boxed{\frac{4\sqrt{6}}{3}}$$

$$\frac{-2x}{\sqrt{10x}}=\frac{-2x\cdot\sqrt{10x}}{\sqrt{10x}\cdot\sqrt{10x}}=\frac{-2x\sqrt{10x}}{\sqrt[2]{10^2x^2}}=\frac{-\cancel{2x}\sqrt{10x}}{\cancel{10x}_{5}}=\boxed{\frac{-\sqrt{10x}}{5}}$$

$$\frac{1}{\sqrt[3]{5}}=\frac{1}{\sqrt[3]{5^1}}=\frac{1\cdot\sqrt[3]{5^2}}{\sqrt[3]{5^1}\cdot\sqrt[3]{5^2}}=\frac{\sqrt[3]{25}}{\cancel{\sqrt[3]{5^3}}}=\boxed{\frac{\sqrt[3]{25}}{5}}$$

$$\frac{2}{\sqrt[5]{4}}=\frac{2}{\sqrt[5]{2^2}}=\frac{2\cdot\sqrt[5]{2^3}}{\sqrt[5]{2^2}\cdot\sqrt[5]{2^3}}=\frac{2\sqrt[5]{8}}{\cancel{\sqrt[5]{2^5}}}=\frac{\cancel{2}\sqrt[5]{8}}{\cancel{2}}=\boxed{\sqrt[5]{8}}$$

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$$\frac{3}{\sqrt{2}-1} = \frac{3(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{3\sqrt{2}+3}{\sqrt{4}+\cancel{\sqrt{2}}-\cancel{\sqrt{2}}-1}$$

$$= \frac{3\sqrt{2}+3}{2-1}$$

$$= \frac{3\sqrt{2}+3}{1} = \boxed{3\sqrt{2}+3}$$

$$\frac{\sqrt{5}}{\sqrt{5}+2} = \frac{\sqrt{5}\,(\sqrt{5}-2)}{(\sqrt{5}+2)(\sqrt{5}-2)} = \frac{\sqrt{25}-2\sqrt{2}}{\sqrt{25}-\cancel{2\sqrt{2}}+\cancel{2\sqrt{2}}-4}$$

$$= \frac{5-2\sqrt{2}}{5-4} = \frac{5-2\sqrt{2}}{1}$$

$$\boxed{5-2\sqrt{2}}$$

$$\frac{2\sqrt{6}}{\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{6}\,(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$$

$$= \frac{2\sqrt{30}-2\sqrt{18}}{\sqrt{25}-\cancel{\sqrt{15}}+\cancel{\sqrt{15}}-\sqrt{9}}$$

$1\cdot18$, $\boxed{2\cdot9}$, $3\cdot6$

$\boxed{1\cdot30,\ 2\cdot15,\ 3\cdot10,\ 5\cdot6}$

$$= \frac{2\sqrt{30}-2\sqrt{9}\sqrt{2}}{5-3}$$

$$= \frac{2\sqrt{30}-6\sqrt{2}}{2} = \frac{2\sqrt{30}}{2} - \frac{6\sqrt{2}}{2}$$

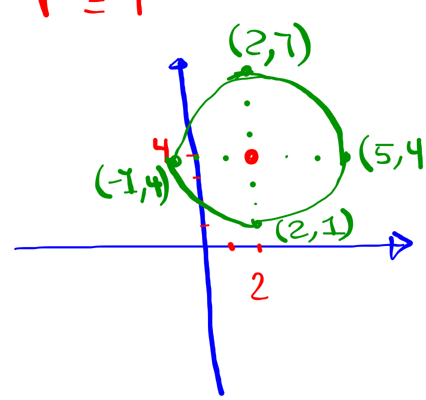
$$= \boxed{\sqrt{30}-3\sqrt{2}}$$

Circle $\quad (x-h)^2 + (y-k)^2 = r^2$

Center (h,k), Radius r

$$(x-2)^2 + (y-4)^2 = 9 \leftarrow r^2 = 9$$

Center $(2,4)$ $\quad r = 3$

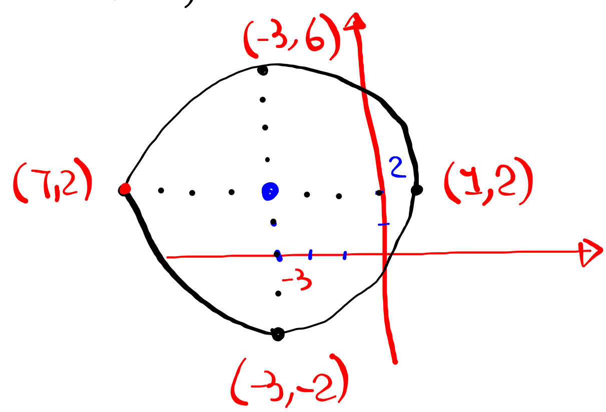


$(x+3)^2 + (y-2)^2 = 16$

$r^2 = 16$

$r = 4$

Center $(-3, 2)$ radius 4



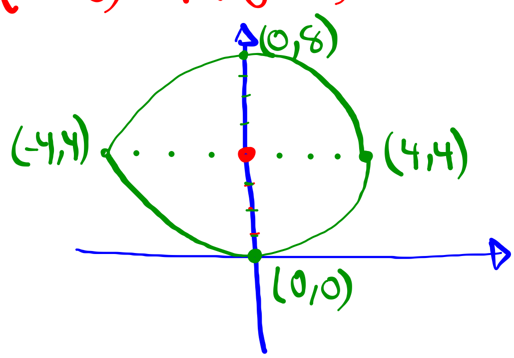
$x^2 + (y-4)^2 = 16$

$(x-0)^2 + (y-4)^2 = 16$

Center $(0,4)$

radius 4

Graph

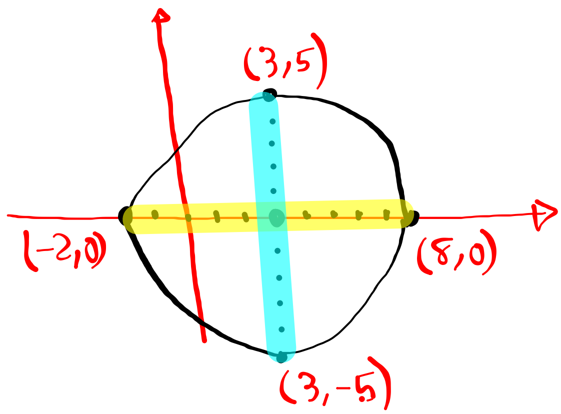


$(x-3)^2 + y^2 = 25$

Center $(3,0)$

radius 5

Draw ✓



Domain $[-2,8]$

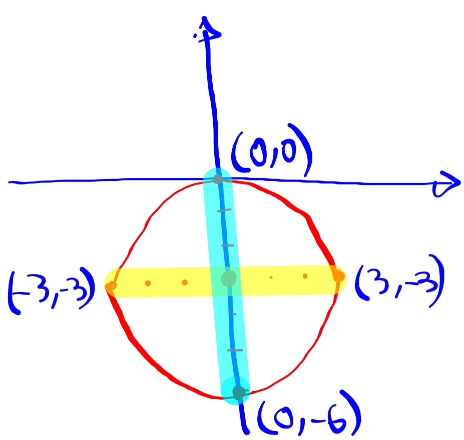
Range $[-5,5]$

$x^2 + (y+3)^2 = 9$

Center $(0,-3)$

Radius 3

Draw



Domain $[-3,3]$

Range $[-6,0]$

Intercepts

x-Int $(0,0)$

Y-Ints $(0,0)$ & $(0,-6)$

Class QZ 16

1) Solve $\sqrt{2x-9} = 5$

2) Solve $x = \sqrt{x^2 - 4x + 4}$